

Quantum Interference: From Kaons to Neutrinos (with Quantum Beats in between)

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ABSTRACT

Using the vehicle of resolving an apparent paradox, a discussion of quantum interference is presented. The understanding of a number of different physical phenomena can be unified, in this context. These range from the neutral kaon system to massive neutrinos, not to mention quantum beats, Rydberg wave packets, and neutron gravity.

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This work is dedicated to the memory of Bernie Deutch, our friend and colleague, whose love of physics was exemplified by his words and by his actions.

1 The Neutral Kaon System

One of the most important “modern, quantum-interference” phenomenon was discovered in particle physics. The background was that, in the 1950’s, it was observed that the strongly-interacting, neutral, K^0 meson, sometimes appeared to decay via the weak interaction into $\pi^+\pi^-$. Mind you, the full quantum field theory of the CPT theorem was not formulated until 1957 [1]. Even so, in 1955 Gell-Mann and Pais [2] predicted an astounding new effect on the basis of these experimental results.

They predicted (in terms of our present terminology) that there must be an antiparticle to the K^0 , called the \bar{K}^0 , with opposite quantum number “strangeness” or “hypercharge.” Further, the origin of the decay to the 2π state is not the pure strangeness eigenstate, but rather the coherent mixture

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) . \quad (1)$$

This state is an eigenstate of CP with eigenvalue $+1$. Another prediction was that there should be an eigenstate of CP with eigenvalue -1 . This state would be

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) . \quad (2)$$

The K_2 would have a slightly different mass than the K_1 , and have a much longer lifetime. Finally, because of this superposition, a beam of particles that was originally composed of K^0 ’s (or \bar{K}^0 ’s) would evolve in time in an interference mode, oscillating between the two decaying species. In particular, inverting Eqs. (1) and (2), inserting the time-dependence, and using the proper time,

$$|K^0(\tau)\rangle = \frac{1}{\sqrt{2}} \left[\exp[-(im_S + \Gamma_S/2)\tau] |K_1\rangle + \exp[-(im_L + \Gamma_L/2)\tau] |K_2\rangle \right] , \quad (3)$$

where the m ’s are the masses and the Γ ’s are the decay widths of the shorter- and longer-lived particles. Similarly for the \bar{K}^0 ’s.

Eventually, all of these predictions were found to be true. The first verification was the discovery of the long-lived K_2 , which decays into a 3π state [3]. But there was a problem in verifying the interference. The K_1 ’s and the K_2 ’s did not decay into the same final states. Therefore, one could not measure $|\langle F|K^0(\tau)\rangle|^2$, for some final state F .

Pais and Piccioni overcame this problem [4] when they realized one could go back and use the strong interactions to remix the particles.

Since the K_1 ’s and K_2 ’s are time-dependent linear combinations of the K^0 ’s and \bar{K}^0 ’s, the reverse is also true. Thus, at any given time, $|K^0(\tau)\rangle$ is not only a linear combination of $|K_1\rangle$ and $|K_2\rangle$, it is also a linear combination of $|K^0\rangle$ and $|\bar{K}^0\rangle$. If this

state is then sent into a “regenerator,” say a slab of copper, the $|K^0\rangle$ forward scattering amplitude, $f(0)$, will be different and smaller than the $|\bar{K}^0\rangle$ forward scattering amplitude, $\bar{f}(0)$. Thus, if the size of the regenerator is varied, the outcoming decay particles will be varying different relative superpositions of $|K_1\rangle$ and $|K_2\rangle$. After a thorough analysis by Good [5], experiments were done, and the effect was seen [6, 7]

These discoveries constituted a beautiful piece of quantum mechanics. However, this was all immediately overshadowed when CP -violation was observed in the K^0 system [8]. Those authors were actually trying to put a better limit on the fact that the K_2 did NOT decay into $\pi^+\pi^-$. But they found, at the level of $\sim 10^{-3}$, that it did. Therefore, the eigenstates of the complete weak system, including CP -violation, are not quite K_1 and K_2 , but rather a slightly different admixture of K^0 and \bar{K}^0 . These “complete” eigenstates are called the K_S and K_L and are parametrized by

$$|K_S\rangle = \frac{1}{[2(1 + |\epsilon|^2)(1/2)]} \left[(1 + \epsilon)|K^0\rangle + (1 - \epsilon)|\bar{K}^0\rangle \right] , \quad (4)$$

$$|K_L\rangle = \frac{1}{[2(1 + |\epsilon|^2)(1/2)]} \left[(1 + \epsilon)|K^0\rangle - (1 - \epsilon)|\bar{K}^0\rangle \right] , \quad (5)$$

$|\epsilon|$ being a number of order 10^{-3} .

However, CP -violation did allow one last piece of quantum mechanics to be done. It meant that a direct interference measurement was possible because both the K_S and the K_L decay into two-pion states. A beam of K^0 's, which from Eqs. (4) and (5) has an equal amplitude (a) for being a K_S and a K_L , will decay into a $\pi^+\pi^-$ state with an intensity proportional to

$$\begin{aligned} I_{\pi\pi}(\tau) &= \frac{|\langle\pi^+\pi^-|K_0(\tau)\rangle|^2}{|P_{+-}|^2} \\ &= \exp[-\Gamma_S\tau] + 2|\eta_{+-}| \exp[-(\Gamma_S + \Gamma_L)\tau/2] \cos(\Delta m\tau - \phi_{+-}) \\ &\quad + |\eta_{+-}|^2 \exp[-\Gamma_L\tau] , \end{aligned} \quad (6)$$

where

$$P_{+-} = \langle\pi^+\pi^-|K_S\rangle , \quad (7)$$

$$\eta_{+-} = |\eta_{+-}|e^{i\phi_{+-}} = \frac{\langle\pi^+\pi^-|K_L\rangle}{\langle\pi^+\pi^-|K_S\rangle} , \quad (8)$$

$$\Delta m = m_L - m_S . \quad (9)$$

This prediction was verified [9]. By 1974 an experiment with 6 million events was able to show the oscillation very clearly [10]. Figure 1 is taken from that paper.

The current experimental numbers of the various parameters are [11]

$$\tau_S = 1/\Gamma_S = (0.8926 \pm 0.0012) \times 10^{-8} \text{ sec} , \quad (10)$$

$$\tau_L = 1/\Gamma_L = (5.17 \pm 0.04) \times 10^{-8} \text{ sec} , \quad (11)$$

$$\Delta m = (0.5333 \times 0.0027) \times 10^{10} \hbar/\text{sec} , \quad (12)$$

$$|\eta_{+-}| = (2.269 \pm 0.023) \times 10^{-3} \approx |\epsilon| \quad (13)$$

$$\phi_{+-} = (44.3 \pm 0.8)^\circ . \quad (14)$$

There are, of course, many excellent discussions of this phenomenon [12, 13]. but in recent times, interest in the CP-violation of the K system has led to less interest in the interference phenomenon of itself [14, 15]. This is understandable since, to this day, CP-violation is not understood from a fundamental theory.

However, the situation is changing, since there are now two other mixing situations which have become of experimental interest. One is that of neutrinos (which I shall return to in Sec. 5) and the other is the B system. With the discovery of higher-mass quarks, it became clear that mixing could reappear in other quark sectors; in particular, in the “beauty” sector of the neutral B mesons. Since the planned B-factory may allow the study of CP-violation in this new system, interest has revived in the quantum interference.

2 The Paradox

The above has led to the formulation of a “paradox” whose resolution, both theoretically as well as by examples of experimental manifestations in other fields, is the focal point of our discussion.

Consider the neutral B^0 - \bar{B}^0 -meson system, whose properties are similar to those of the K -meson system discussed in the last section. Ignoring possible CP-violation and differences in lifetimes, one could describe the two “beauty” eigenstates as combinations of the two mass eigenstates, $|B_L\rangle$ and $|B_H\rangle$, which have masses M_L and M_H . Specifically,

$$|B^0\rangle = \frac{1}{\sqrt{2}}(|B_L\rangle + |B_H\rangle) , \quad (15)$$

$$|\bar{B}^0\rangle = \frac{1}{\sqrt{2}}(|B_L\rangle - |B_H\rangle) . \quad (16)$$

An argument has been presented [16] that it is contradictory to try to discuss a beam of B particles oscillating where the components have different energies. The argument is that only by looking at the positions of the components as functions of time, and interpreting them as momenta, can one properly describe things.

In particular, consider a B^0 produced at $x = 0$ in a state of definite energy, E . The momenta of the B_L and B_H components, p_L and p_H , are given by

$$p_L^2 = E^2 - M_L^2 , \quad p_H^2 = E^2 - M_H^2 . \quad (17)$$

Then, *as a function of x* , $B^0(x)$ will have a relative mixture of \bar{B}^0 to B^0 of

$$\begin{aligned} \left| \frac{\langle \bar{B}^0 | B^0(x) \rangle}{\langle B^0 | B^0(x) \rangle} \right|^2 &= \left| \frac{e^{ip_L x} - e^{ip_H x}}{e^{ip_L x} + e^{ip_H x}} \right|^2 = \tan^2 \left(\frac{(p_L - p_H)x}{2} \right) \\ &= \tan^2 \left(\frac{(M_L^2 - M_H^2)x}{2(p_L + p_H)} \right). \end{aligned} \quad (18)$$

This is the normal $B - \bar{B}$ oscillation result.

Next the discussion [16] considers the case where a B^0 is produced at time $t = 0$ in a state of definite momentum, p . The energies of the B_L and B_H components, E_L and E_H , are

$$E_L^2 = p^2 + M_L^2, \quad E_H^2 = p^2 + M_H^2. \quad (19)$$

Then, as a function of time, $|B^0(t)\rangle$ will have relative components of $|\bar{B}^0\rangle$ to B^0 given by

$$\begin{aligned} \left| \frac{\langle \bar{B}^0 | B^0(t) \rangle}{\langle B^0 | B^0(t) \rangle} \right|^2 &= \left| \frac{e^{iE_L t} - e^{iE_H t}}{e^{iE_L t} + e^{iE_H t}} \right|^2 = \tan^2 \left(\frac{(E_L - E_H)t}{2} \right) \\ &= \tan^2 \left(\frac{(M_L^2 - M_H^2)t}{2(E_L + E_H)} \right) \end{aligned} \quad (20)$$

$$\approx \tan^2 \left(\frac{(M_L^2 - M_H^2)x}{4p} \right), \quad (21)$$

where the last approximate equality is obtained by using

$$x = vt = \frac{p}{E} \cdot t. \quad (22)$$

Eqs. (18) and (21) are the same result, so there seems to be no problem.

Contrariwise, the argument is raised [16], if the states have different momenta do they not also have different velocities, v_L and v_H , so that they therefore arrive at the point x at different times, t_L and t_H ?

$$x = v_L t_L = \frac{p}{E_L} \cdot t_L = v_H t_H = \frac{p}{E_H} \cdot t_H. \quad (23)$$

Then the time-dependence of $|B^0\rangle$ and $|\bar{B}^0\rangle$ as a function of x would be

$$\left| \frac{\langle \bar{B}^0 | B^0(x) \rangle}{\langle B^0 | B^0(x) \rangle} \right|^2 = \left| \frac{e^{iE_L t_L} - e^{iE_H t_H}}{e^{iE_L t_L} + e^{iE_H t_H}} \right|^2 = \tan^2 \left(\frac{(E_L t_L - E_H t_H)}{2} \right)$$

$$= \tan^2 \left(\frac{(M_L^2 - M_H^2)x}{2p} \right) . \quad (24)$$

The comparison of Eq. (24) with Eq. (21) is the paradox. There appears to be an inconsistency in how to make a superposition of different energy eigenstates. One might even argue that this ambiguity implies that one should not consider interference between states of different energies. Since this is commonly done in the K system by going to the center-of-mass system and using the proper time, τ , one would have to further explain why a standard use of special relativity is not valid.

To understand the resolution of this paradox, one can return to the classic quantum-mechanical interference problem, the double slit experiment. If you know which slit the electron goes through, then you lose the interference pattern. If you know what arm of an interferometer the “particle” goes through, you lose the interference. This last has been shown to be true even in “delayed choice” experiments, where the decision to find out which arm the particle is in is made *after* the particle has entered the interferometer [17].

The same is true here. With interference, these are *not* individual particles. They are components in a mixed state. If you know *which* of the two pure states, $|B_L\rangle$ or $|B_H\rangle$, you have, then you lose the interference and the interference pattern disappears. In Sec. 5 we will point out where this result has been made mathematically precise in the context of massive neutrino propagation.

3 Quantum Beats

At this point it is illuminating to show cases where there is well-known and understood interference of states with different energies.

A very clear example is that of “quantum beats,” in atomic and molecular physics. As reviewed in Refs. [18, 19, 20], quantum beats were first demonstrated in 1964 without the use of lasers. Since then, the use of lasers has allowed the detection of quantum beats in Zeeman and hyperfine structures of many atoms and molecules. The example I give here is from a molecule, S_1 propynal ($HC \equiv CCHO$), because it exhibits an interference structure analogous to the K -meson system.

Consider the four-level system shown in Figure 2. From the ground state $|g\rangle$ a pair of closely-spaced excited states $|a\rangle$ and $|b\rangle$ are excited by a short laser pulse of appropriate frequency and bandwidth larger than the energy splitting of the excited states. The coherent superposition of the two excited states at $t = 0$, when the laser stops, is given by

$$|\psi(t=0)\rangle = \mu_{ga}|a\rangle + \mu_{gb}|b\rangle , \quad (25)$$

where μ denotes a dipole matrix element:

$$\mu_{jk} = \langle k|d|j\rangle . \quad (26)$$

The time development of this state is now given by the frequencies

$$\omega_a = E_{ga}/\hbar , \quad \omega_b = E_{gb}/\hbar , \quad (27)$$

and the decay constants γ_a and γ_b :

$$|\psi(t)\rangle = \mu_{ga} \exp[-(\omega_a + \gamma_a/2)t] |a\rangle + \mu_{gb} \exp[-(\omega_b + \gamma_b/2)t] |b\rangle . \quad (28)$$

Then the intensity of photons to the final state $|f\rangle$ will be proportional to

$$I_d(t) = |\langle f | d | \psi(t) \rangle|^2 \quad (29)$$

$$= |\mu_{ag}|^2 |\mu_{fa}|^2 \exp[-\gamma_a t] + |\mu_{bg}|^2 |\mu_{fb}|^2 \exp[-\gamma_b t] \\ + 2 |\mu_{ag} \mu_{fa} \mu_{bg} \mu_{fb}| \exp[-(\gamma_a + \gamma_b)t/2] \cos[(\omega_a - \omega_b)t + \theta] . \quad (30)$$

In Figure 3 we show the results from an experiment using S_1 propynal [18, 20]. The main oscillation follows that of Eq. (30), which exhibits the form of Eq. (6).

In fact, there is another aspect to this system that is quite cute. In actuality, this systems is composed of two sets of two coherently excited levels, with quantum beat frequencies of 16.6 MHz and 16.8 MHz. Therefore, there is a "beat" of the "beat frequencies," at 200 kHz. That is seen in the long period oscillation which occurs at about 2 μ s in Figure 3.

But the point to be made is that these are different energy states clearly exhibiting quantum interference when the position and momentum of the states has nothing to do with the description.

4 Other Phenomena

4.1 Rydberg wave packets

During the past decade, a very interesting phenomenon has been studied, Rydberg wave packets [21, 22, 23, 24]. A short-pulsed laser beam is used to excite a mixed state with high- $\langle n \rangle$. This packet has a significant overlap with a number of eigenstates of different energies (and in fact is a squeezed state [25]). That these packets exhibit classical motion and follow a classical Kepler orbit is deduced from the following argument:

One measures the rate at which the atoms decay from their excited energy packets. There is an oscillation in the number of decays per unit time, and the oscillation period is that which would obtain for an elliptical Kepler orbit of that energy. If the particle is in an elliptical Kepler orbit, then it would undergo more acceleration near the perigee of the orbit vs. near the apogee. Since the rate of decay increases with acceleration, one infers that the oscillations reflect a coherent wave-packet being near the perigee (more decays) and then heading towards the apogee (fewer decays).

Here is an example of a coherent wave-packet composed of eigenstates of many energies. It has a spatial coherence and follows a "classical-like" Kepler orbit.

4.2 Neutron interferometry and the COW experiment

Another interesting experiment was the COW experiment [26]. Here individual neutrons were sent through a single-crystal neutron interferometer. The interferometer could be rotated about an axis defined by the incoming neutron beam. Therefore,

upon entering the interferometer, one part of the neutron beam would rise to a higher position (and hence gravitational potential) than the other. The two split beams would propagate, and then at the end the second beam would be brought up to interfere with the first. Depending upon the height difference, the interference-pattern fringes shifted and hence yielded a measurement of local g on the neutrons.

The interesting thing for us here is that a complete analysis can and has been given by Greenberger and Overhauser, using both the spatial and time dependencies, in the frameworks of Galilean relativity, special relativity, and general relativity, and in the lab and freely falling frames [27]. There are no inconsistencies in the interpretations and they agree with the experimental results.

5 Neutrino Oscillations

The last topic, that of neutrino oscillations, is the most timely. They may have been observed at the LAMPF beam stop. The LSND group [28] has just reported the observation of electron-antineutrino events from a beam of incident muon antineutrinos obtained by the decay of first π^+ 's and then μ^+ 's. This may explain the paucity of neutrinos in solar neutrino experiments [29]. There, for example, one might see only half the expected electron neutrinos because the other half are muon neutrinos by the time they reach the earth.

This puts even more interest in facilities like the proposed Super Nova Burst Observatory [30]. This is a proposed underground laboratory at the WIPP site in New Mexico. Neutron counters would observe the signal from all flavors of incident neutrinos (including muon and tau neutrinos). They would be produced by the neutral-current interaction on nuclei. The ordinary charged-current interaction of the electron neutrinos would give a “massless” time-of-arrival signal at other laboratories, such as Kamiokande. If the other neutrinos had a significant non-zero mass, then their times-of-arrivals would be delayed by the factor

$$\delta t = 5.14 \times 10^{-3} R_{kpc} \left[\left(\frac{m_1}{E_1} \right)^2 - \left(\frac{m_2}{E_2} \right)^2 \right] , \quad (31)$$

where R_{kpc} is the distance to the supernova in kiloparsecs and the m 's and E 's are the masses and energies of two distinct neutrino species,

But this brings us back to our paradox, where the two B mesons were interfering and yet had distinct arrival times. Kayer has analyzed the massive neutrino scenario exhaustively [31]. He showed that, if one takes into account the fact that the beam is composed of wave packets of finite size, not infinite plane waves, when the wave packets no longer overlap one can determine the individual arrival times and the interference pattern is gone.

6 Conclusion

To summarize, quantum mechanics gives us a choice. We can observe interference or we can tell which path or where or what particle we have. But if we have the latter,

then the interference is gone.

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Figure Captions

FIGURE 1. Taken from Figure 4 of Ref. [10]. It shows the time distribution of $K \rightarrow \pi^+\pi^-$ events. (a) Events (*histogram*) and fitted distribution (*dots*). (b) Events corrected for detection efficiency (*histogram*), fitted distribution with interference term (*dots*), and without interference term (*curve*). Insert: Interference term as extracted from data (*dots*) and fitted term (*line*).

FIGURE 2. Taken from Figure 1 of Ref. [18]. A four-level system. . States $|a\rangle$ and $|b\rangle$ are coherently excited from a single ground state $|g\rangle$. The coherence is evidenced by an interference effect (quantum beat) when the emission decay to a common final state $|f\rangle$ is observed.

FIGURE 3. Taken from Figure 2a of Ref. [18]. Quantum beats in the fluorescence decay of the S_0^1 band of S_1 propynal. Shown is the time domain signal.

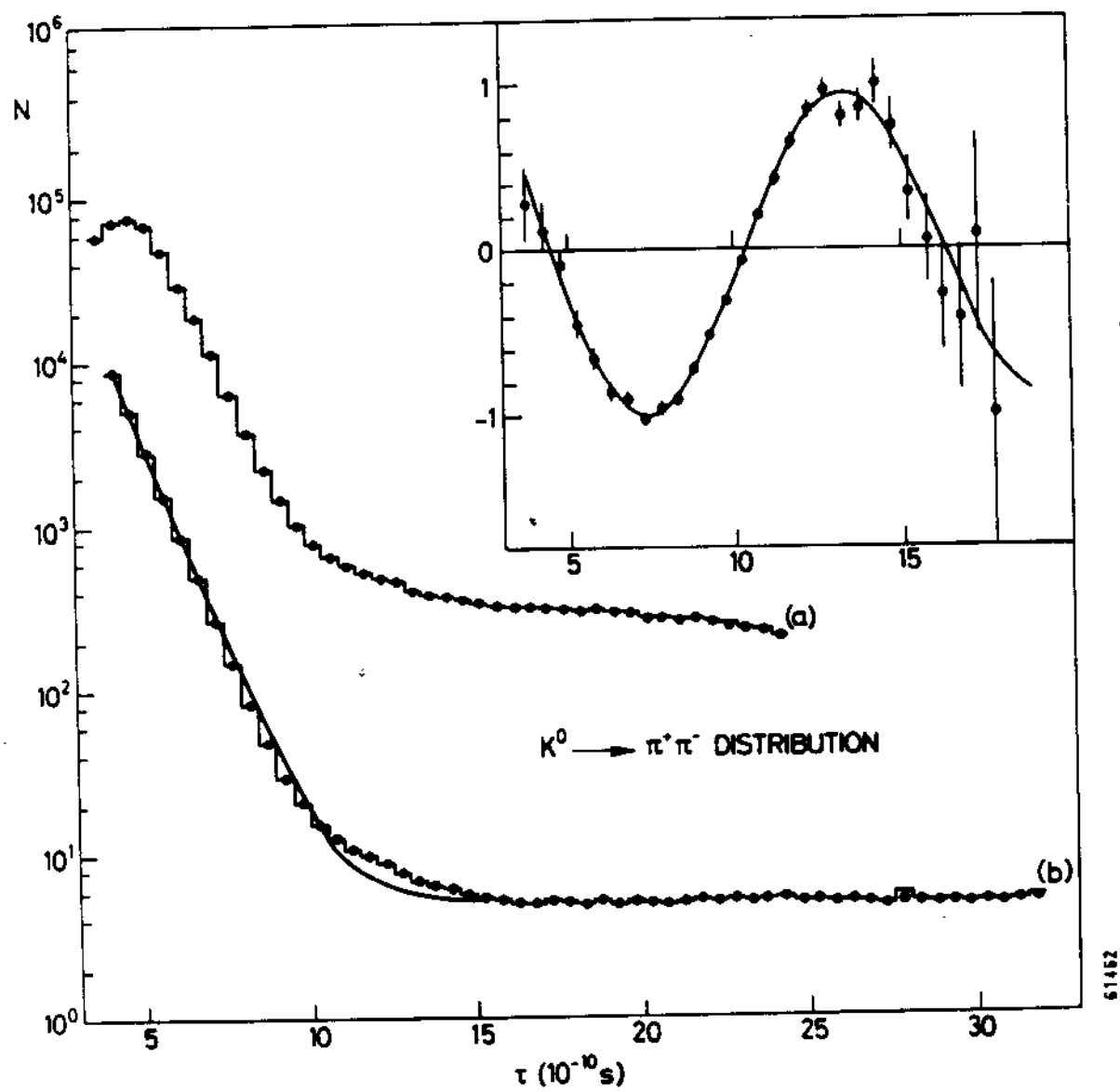


Figure 1

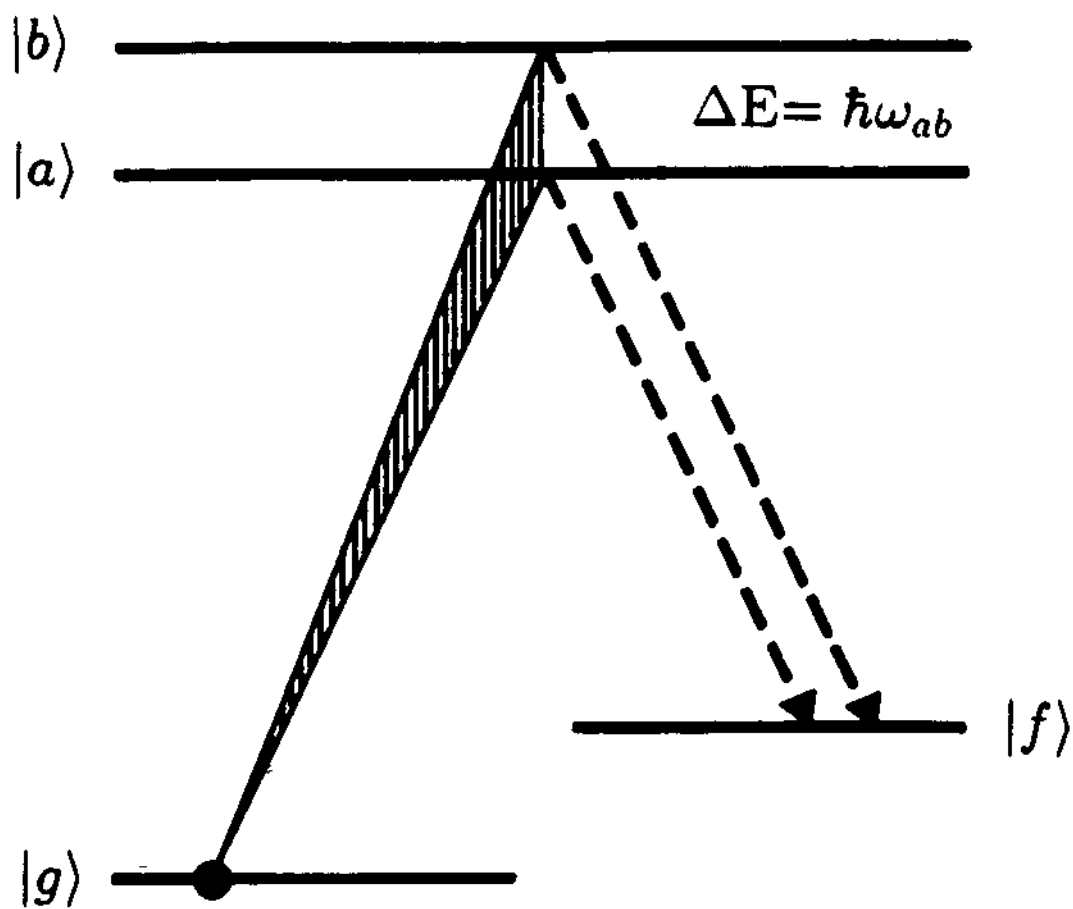


Figure 2

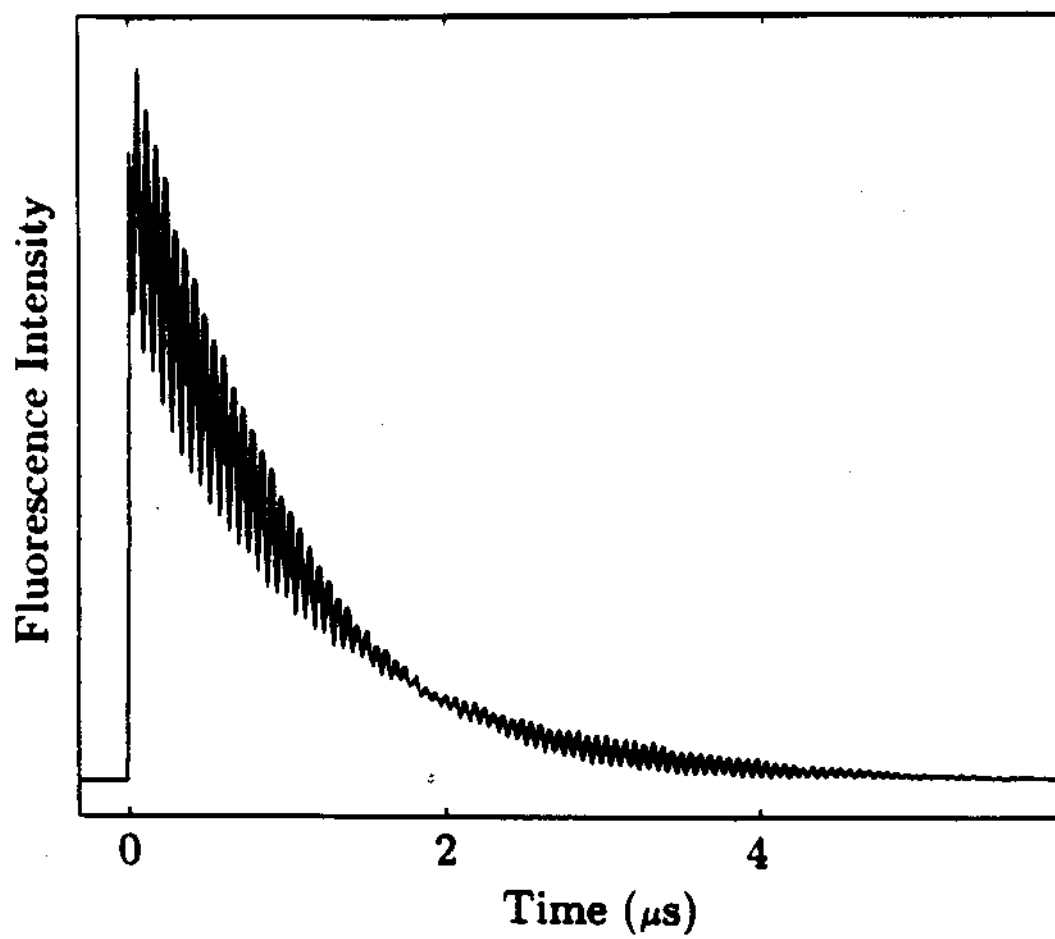


Figure 3